Lecture 2: Review of Signal Analysis

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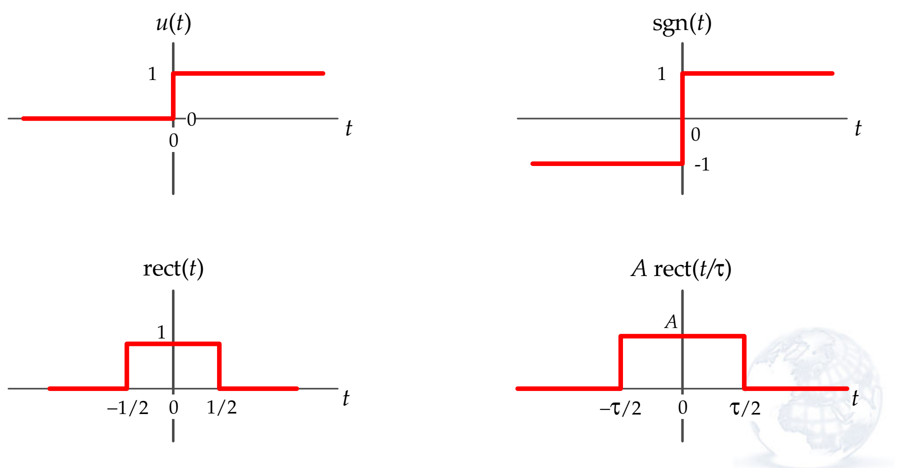
Basic Signals



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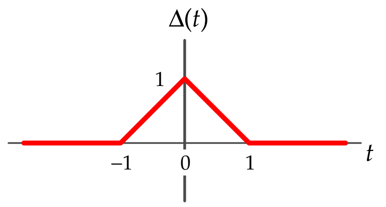

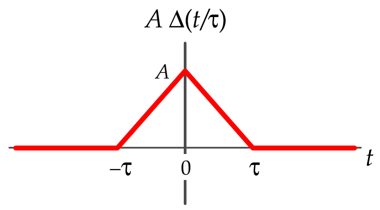

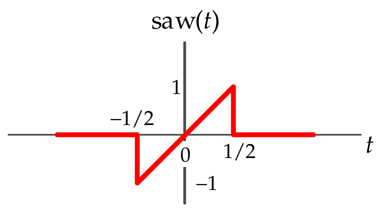

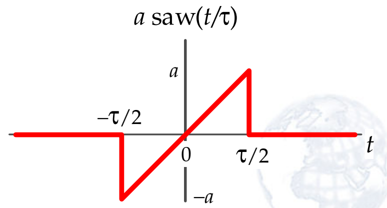

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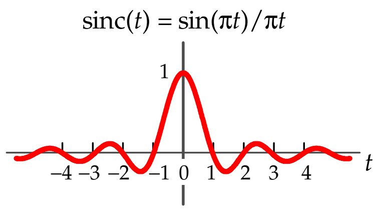
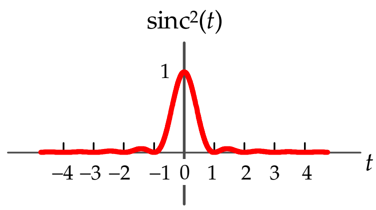
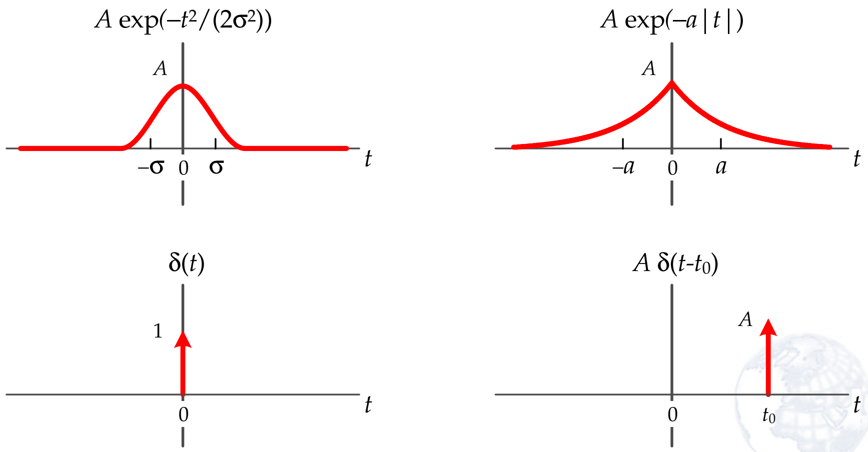




$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\cancel{\operatorname{sinc}(t) = \operatorname{Sa}(t) = \frac{\sin(t)}{t}}$$

Basic Signals



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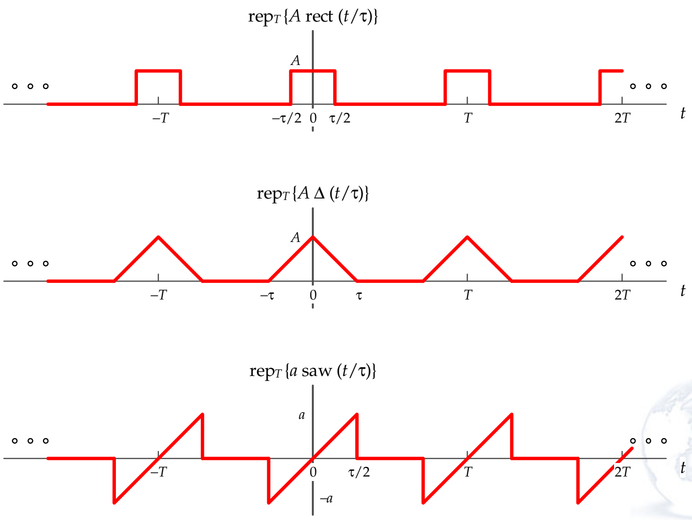
Periodic Signals



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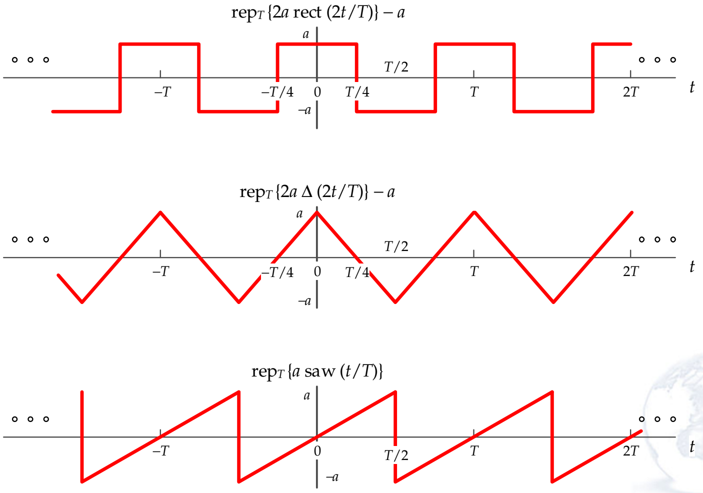
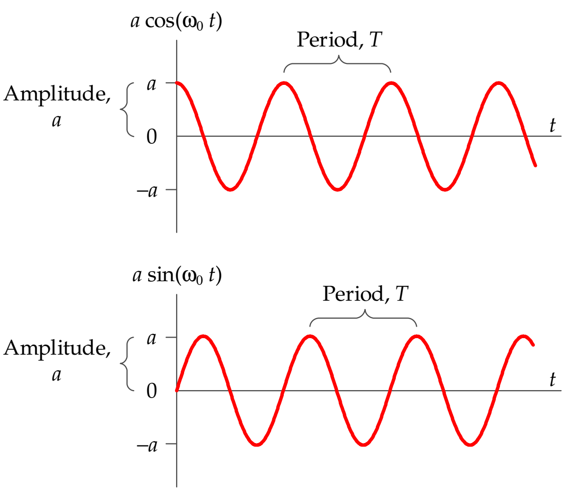
Periodic Signals



Periodic Signals



Exponential/Trigonometric/Compact

$$x(t) = \sum_{n=-\infty}^{\infty} \alpha_n \cdot e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right), \quad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t - \theta_n), \quad \omega_0 = \frac{2\pi}{T}$$

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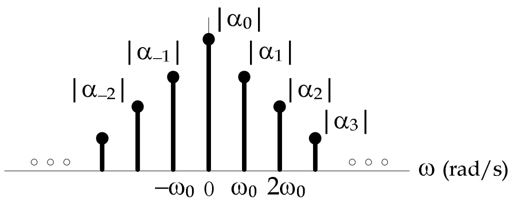
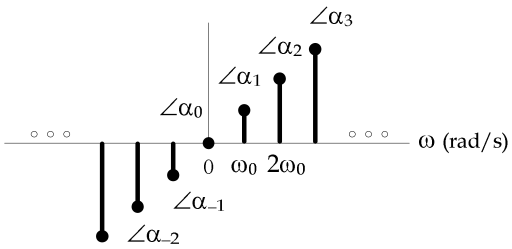
Exponential Fourier Series

Magnitude Spectrum



Phase Spectrum



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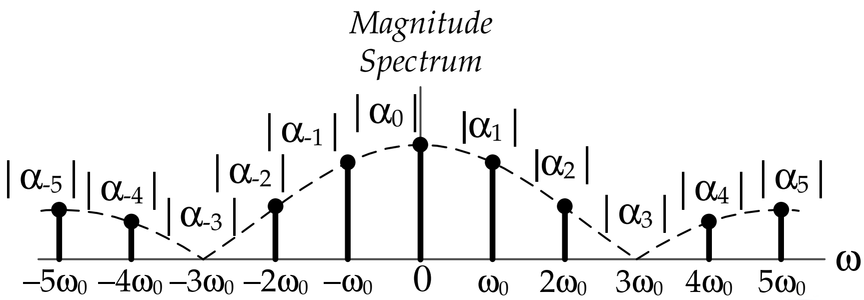
First-Null Bandwidth



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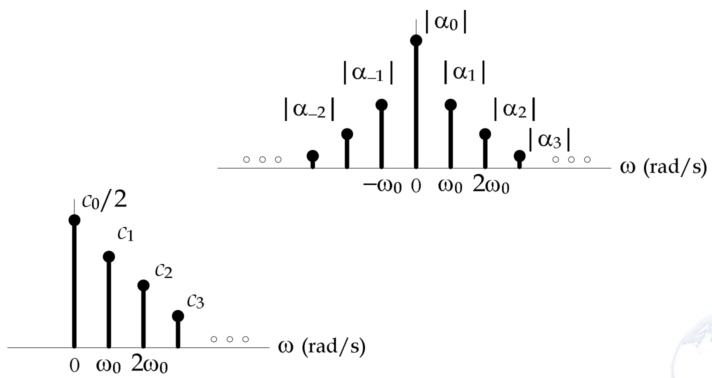
Single-Sided vs. Double-Sided



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Fourier Transform

$$X(\omega) = \mathscr{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \mathscr{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

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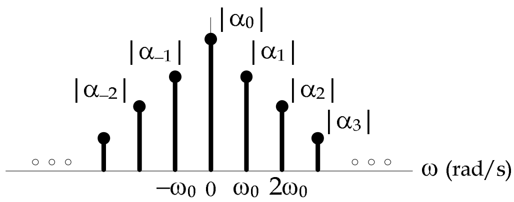
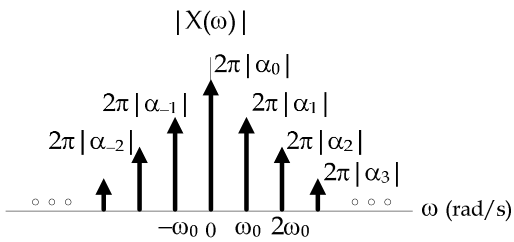
Fourier Series vs. Tranform





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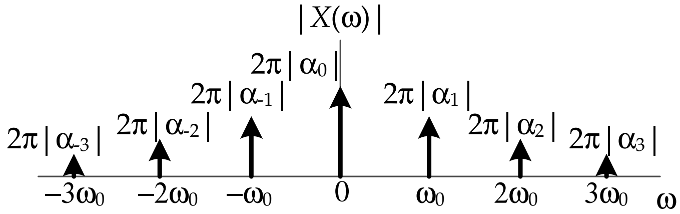
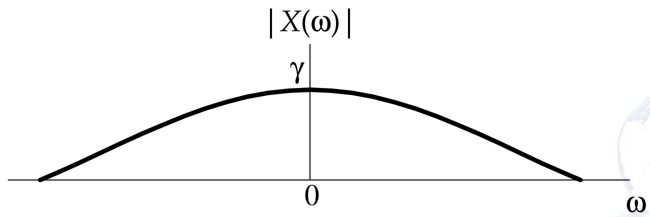
Periodic vs. Aperiodic



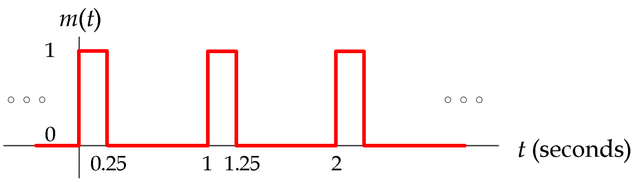


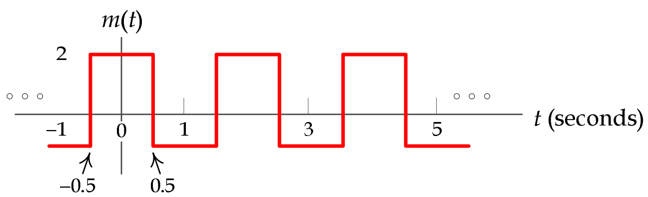
$x(t)$	$X(\omega) = \mathscr{F}\{x(t)\}$
$a\cos(\omega_0 t)$	$\pi a\delta(\omega-\omega_o)+\pi a\delta(\omega+\omega_o)$
$a\sin(\omega_0 t)$	$-j\pi a\delta(\omega-\omega_o)+j\pi a\delta(\omega+\omega_o)$
$e^{\pm j\omega_0 t}$	$2\pi\delta(\omega\mp\omega_0)$
$\mathrm{rect}(t)$	$\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$
$\mathrm{rect}\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
$\Delta(t)$	$\mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$
$\Delta\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$
$\mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$	$\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$
$\mathrm{sinc}\left(\frac{t}{2\pi}\right)$	$2\pi\ \mathrm{rect}(\omega)$
$\mathrm{saw}\left(\frac{t}{\tau}\right)$	$\frac{-2j}{\omega}\left[\mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)-\cos\left(\frac{\omega\tau}{2}\right)\right]$
$\delta(t)$, Dirac delta function	1
1	$2\pi\delta(\omega)$
$\mathrm{rep}_T\{p(t)\}$, periodic	$\sum_{n=-\infty}^{\infty}2\pi\alpha_n\delta(\omega-n\omega_o)$
$\mathrm{rep}_T\{\delta(t)\}=\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\sum_{n=-\infty}^{\infty}\frac{2\pi}{T}\delta(\omega-n\omega_o)=\omega_0\mathrm{rep}_{\omega_0}\{\delta(\omega)\}$
$\mathrm{rep}_T\left\{A\ \mathrm{rect}\left(\frac{t}{\tau}\right)\right\}$	$\sum_{n=-\infty}^{\infty}2\pi\frac{A\tau}{T}\mathrm{sinc}\left(\frac{n\omega_o\tau}{2\pi}\right)\delta(\omega-n\omega_o)$
$\mathrm{rep}_T\left\{A\ \Delta\left(\frac{t}{\tau}\right)\right\}$	$\sum_{n=-\infty}^{\infty}2\pi\frac{A\tau}{T}\mathrm{sinc}^2\left(\frac{n\omega_o\tau}{2\pi}\right)\delta(\omega-n\omega_o)$
$\mathrm{rep}_T\left\{a\ \mathrm{saw}\left(\frac{t}{T}\right)\right\}$	$\sum_{n=-\infty}^{\infty}j2\pi a\frac{\cos(n\pi)}{n\pi}\delta(\omega-n\omega_o)$
$u(t)$, unit step function	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\mathrm{sgn}(t)=u(t)-u(-t)$	$\frac{2}{j\omega}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\ e^{-\sigma^2\omega^2/2}$
$e^{-a\lvert t\rvert},\quad a>0$	$\frac{2a}{a^2+\omega^2}$

Fourier Transform Properties

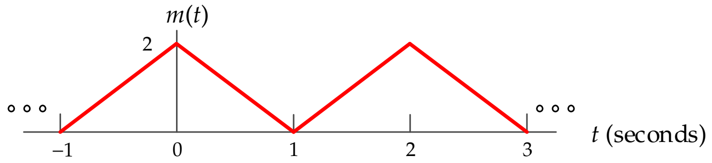
Property	$x(t)$	$X(\omega) = \mathscr{F}\{x(t)\}$
Linearity (superposition)	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
Complex conjugate	$x^*(t)$	$X^*(-\omega)$
Symmetry	$x_{even}(t)$ $x_{odd}(t)$	$X_{even}(\omega)$, real $X_{odd}(\omega)$, imaginary
Duality	$X(t)$	$2\pi x(-\omega)$
Time scaling (reciprocal spreading)	$x\left(\frac{t}{\tau}\right)$	$\lvert\tau\rvert\ X(\tau\omega)$
Time inversion (time reversal)	$x(-t)$	$X(-\omega)$
Time shift (time delay/advance)	$x(t \pm t_0)$	$X(\omega)e^{\pm j\omega t_0}$
Frequency shift	$x(t)e^{\pm j\omega_0 t}$	$X(\omega \mp \omega_0)$
Modulation	$x(t)\cos(\omega_0 t) =$ $\frac{x(t)}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$	$\frac{1}{2}X(\omega - \omega_0) + \frac{1}{2}X(\omega + \omega_0)$
Time differentiation	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^n\ X(\omega)$
Time integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(\omega)}{j\omega} + \pi X(0)\ \delta(\omega)$
Time convolution	$x(t) \circledast y(t)$	$X(\omega)Y(\omega)$
Frequency convolution	$x(t)y(t)$	$\frac{1}{2\pi}\left(X(\omega) \circledast Y(\omega)\right)$

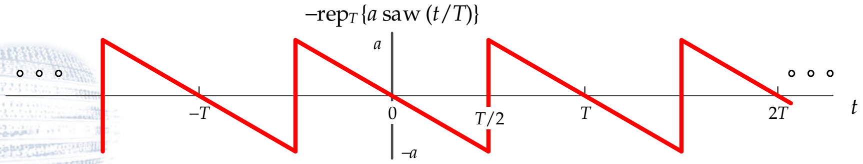
Practice Problems





Practice Problems





DC vs. Average Power

The DC value or average value of the signal $x(t)$ is:

$$DC = \overline{x(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt$$

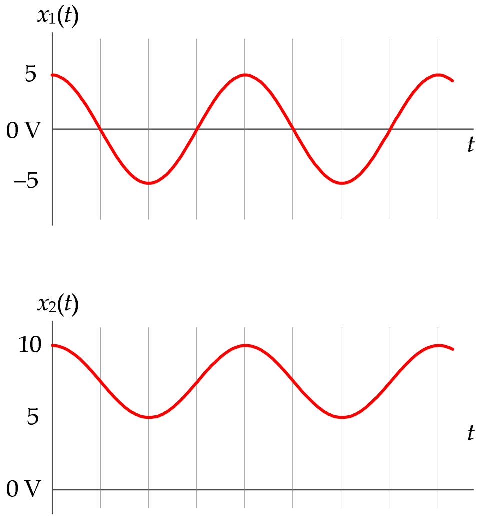
$$DC = \overline{x(t)} = \alpha_0$$

The average power in the signal $x(t)$ is:

$$P_x = \overline{x^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P_x = \overline{x^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)d\omega$$

DC vs. Average Power



Orthogonality

$$4\cos(100t) + 2\cos(300t)$$

$$5\cos(100t) + 3\cos(600t - 30°)$$

$$5\cos(100t) + 7\sin(400t)$$

$$2\cos(100t) + 3\cos(100t - 40°)$$

$$2\cos(100t) + 3\sin(100t)$$

Orthogonality

$$2\cos(100t) + 3\cos(100t)$$

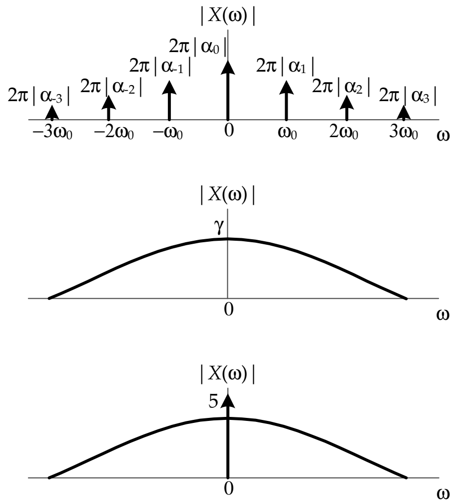
$$4\cos(100t) + 7$$

$$3\cos(100t) + 2\cos(400t) + 6$$

$$\text{rep}_T\{2A\,\text{rect}(2t/T)\} - A/2$$

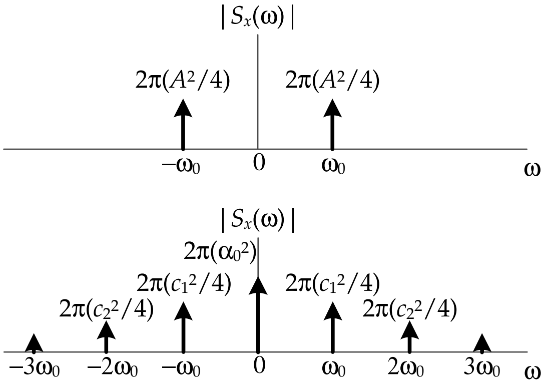
$$\text{rep}_T\{2A\,\text{rect}(2t/T)\} - A$$

DC from Frequency Domain



Power Spectral Density

$$\text{PSD} = S_x(\omega) = \lim_{T\to\infty} \frac{1}{T} |X_T(\omega)|^2 = \mathcal{F}\{R_{xx}(\tau)\}$$

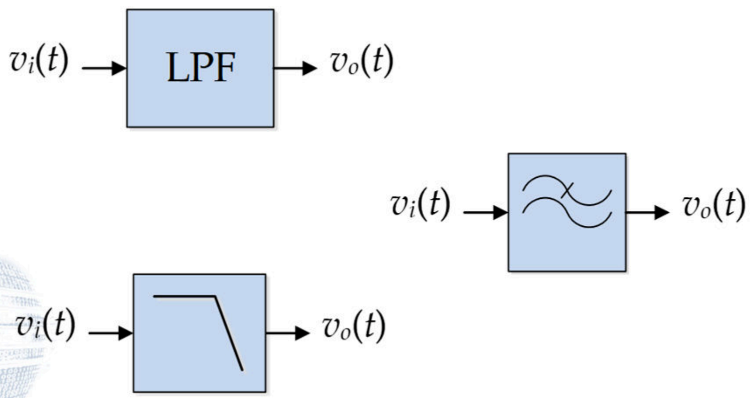


Quick Review of Filters

- There are four main filter types that you studied in signal analysis:
  - **LPF: Low-Pass Filter**
  - **BPF: Band-Pass Filter**
  - HPF: High-Pass Filter
  - BSF: Band-Stop Filter / Notch Filter.

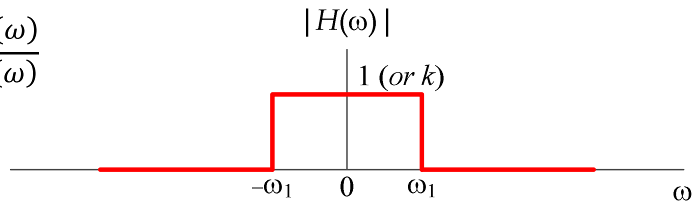
Low-Pass Filter (LPF)

- Symbol:

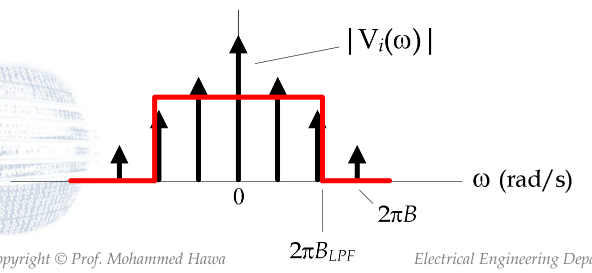


Frequency Response Function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



$|V_o(\omega)| = |H(\omega)| \times |V_i(\omega)|$

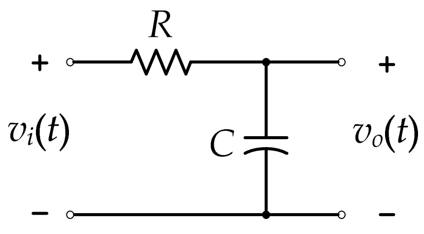


Characteristics/Specifications

- Always *centered* at 0 rad/s.
- *Bandwidth* =
  Cutoff frequency =
  Corner frequency =
  -3 dB frequency =
  ω_1 rad/s
- *Gain* = k.

w/ Bandwidth = 5 kHz
& Gain = 2

Example Circuit

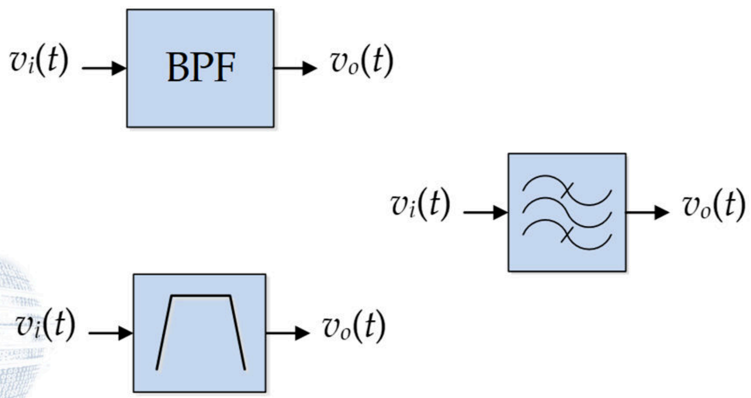


$$B_{LPF} = \frac{1}{2\pi RC}\ \text{Hz}$$

$$Gain = 1$$

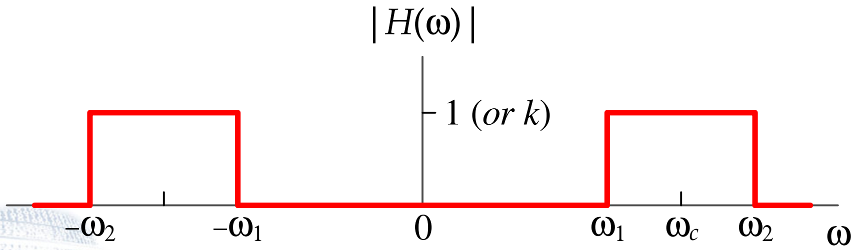
Band-Pass Filter (BPF)

- Symbol:



Frequency Response Function

$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)}$$

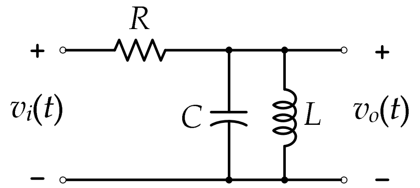


Characteristics/Specifications

- Centered around center frequency ω_c rad/s.
- Bandwidth of filter = $\omega_2 - \omega_1$ rad/s
- Gain = k.

w/ Bandwidth = 80 kHz
Center Frequency = 100 MHz
& Gain = 1

Example Circuit



$$f_c = f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$B_{BPF} = \Delta f = \frac{R}{2\pi L} \text{ Hz}$$

$$Gain = 1$$